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13. ABSTRACT (Maximum 200 words) We report on experiments with strings of laser-cooled Ca^+ ions for quantum information processing. An experiment using the isotope $^{43}\text{Ca}^+$ was set up from scratch. We devised techniques for loading and laser-cooling $^{43}\text{Ca}^+$ ion strings, initializing the ions in the hyperfine clock states and reading out the quantum state with high efficiency. Coherence times of the qubits are about 1s. The single-qubit gates were implemented using microwave and Raman excitation, and coupling to the motional state was demonstrated. Very recently, a pair of $^{43}\text{Ca}^+$ ions were entangled for the first time. In a second line of experiments, we demonstrated simple quantum algorithms with few $^{40}\text{Ca}^+$ ions as well as the implementation of a high-fidelity gate operation entangling a pair of ions with a fidelity of 99.3(1)%. Among the milestone experiments are the demonstration and quantum process tomography of a Cirac-Zoller gate with mean fidelity of about 93%, the demonstration of entanglement in a decoherence-free subspace lasting for 20s, the demonstration of deterministic quantum teleportation, a Toffoli gate, deterministic entanglement swapping a partial measurement of a GHZ state preserving entanglement in the remaining qubits, creation and analysis of an 8-ion W-state, the application of entangled ions for precision spectroscopy.				
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Enclosure 1

Final Progress Report

**Quantum Information Processing
with Trapped $^{43}\text{Ca}^+$ Ions**

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3. Statement of the problem studied

Laser-cooled trapped ions are ideally suited to investigate and implement quantum information processing as they exhibit unique properties: localization of the single particle to less than a few ten nanometers, control of the motional state down to the zero-point of the trapping potential, a high degree of isolation from the environment and thus a very long time available for manipulations of their quantum state. The very same properties make single trapped ions well suited for storing quantum information in long-lived internal states. Together with the ability to detect the ion's quantum state with high precision via the electron shelving technique, trapped ions are a promising approach to quantum computation. Therefore, a number of groups has started a research program to experimentally explore the feasibility of quantum information processing with trapped ions after the seminal proposal by I. Cirac and P. Zoller in 1995.

Processing quantum information with trapped ions relies on the ability to reliably store the quantum information in internal states of the ions having a long lifetime. There are two different ways of encoding a quantum bit in a single ion: (1) Using a ground state in combination with a long-lived metastable excited state results in an *optical qubit* that can be manipulated by single-photon transitions driven by a narrow-band laser. (2) A qubit encoded in two hyperfine ground states (or two Zeeman ground states) is called a hyperfine (or Zeeman) qubit that can be manipulated by Raman two-photon transitions driven by a pair of laser beams with a frequency difference in the range of a few GHz (or MHz in the case of Zeeman states).

The latter approach has been pursued by a number of research groups, most notably the group of D. Wineland in Boulder: In their experiments they use trapped Be^+ ions in a very tightly confining ion trap which allows them to operate at very high trap frequencies [Tur00, Win98]. This results in comparatively easy ground state cooling procedures since the motional sidebands are widely separated and spectroscopically well accessible. On the other hand, the tight confinement in their experiment leads to very small inter-ion distances, which, in turn, renders addressing of individual ions by optical means very difficult. However, using miniaturized segmented traps allows for overcoming some of these difficulties.

The experiments being performed at Innsbruck till the start of this grant were all using optical qubits encoded in the $S_{1/2}$ ground state and the $D_{5/2}$

metastable state of $^{40}\text{Ca}^+$ ions. The experiments have been designed to allow for individual optical access to the qubits provided by the ion string. Thus, the experiments operate in a regime quite different from the Boulder approach. In fact, with our system it is possible to access and manipulate the quantum information carried by the individual ions in a straightforward and easy way. The trade-off, however, is that the weaker axial potential used in our setup results in slower two-qubit gate speeds. At the time when our application for the grant was approved, the status of the experiment was the following:

We had demonstrated a system of qubits (two ions at that time) that each could be initialized with a fidelity of about 98%. The quantum state of a qubit could be correctly identified with a probability of about 98-99%. Qubit coherence times were on the order of 1.5 ms. Single qubits could be coherently manipulated with a fidelity of approximately 99%. In the presence of a second ion, this number decreased as the second ion was excited by spurious laser light, the addressing error (the ratio of Rabi frequencies on the second and the first ion) being about 7%. An entangling gate had been demonstrated that closely followed the proposal by I. Cirac and P. Zoller. Using this gate operation, we were able to entangle two ions with a fidelity of about 71%.

Some of the key experimental barriers were

- qubit dephasing through uncontrolled Zeeman-shifts due to ambient magnetic field fluctuations
- qubit phase error caused by uncontrolled AC-Stark shift during qubit manipulations
- off-resonant excitation leading to bit-flip errors
- residual short-time intensity and frequency instability of the laser source driving the qubit transition
- slow drift of laser intensity and frequency
- limited resolution and switching time of addressing optics

The barriers were partly of technical nature, such as the laser drifts and instabilities, partly they had a physical origin. For this reason, the goals of the grant application were three-fold:

- to improve the existing experiments by overcoming technical problems limiting the qubit coherence and quality of coherent manipulation of the system,
- to demonstrate an improved quantum memory as well as more complex coherent entangling operation in a system of a few ion-qubits and to develop tools for characterizing the coherent operations,
- to set up a new experiment using the isotope $^{43}\text{Ca}^+$ in order to take advantage of the longer coherence times possible with this isotope as compared with $^{40}\text{Ca}^+$,
- to start the development of segmented ion traps in order to handle larger number of ions.

4. Summary of the most important results

4.1 Setting up a new ion-trap experiment with trapped $^{43}\text{Ca}^+$ ions

Processing quantum information with trapped $^{43}\text{Ca}^+$ ions offers the prospect of achieving long coherence times by storing quantum information in the $F=4, m_f=0$ and $F=3, m_f=0$ hyperfine states of the $S_{1/2}$ ground state which are independent of the first order Zeeman effect at zero magnetic field. For the magnetic fields that are typically applied in order to resolve the Zeeman states, the field dependence is still much weaker than for any state in the isotope $^{40}\text{Ca}^+$. Moreover, manipulation of the qubit can be achieved using a pair of laser beams with a frequency difference of about 3.2 GHz for driving Raman transitions between the hyperfine ground states. This considerably relaxes the requirements for laser frequency stability as the qubit phase is no longer referenced to the absolute phase of the laser field driving the transition but only to the difference of laser phases of the Raman beams.

To start working with $^{43}\text{Ca}^+$ required setting up a whole new experiment. The infrastructure (lab space, optical tables, general lab equipment) was provided by the Institute for Quantum Optics and Quantum Information. Using the grant, we were able to design a new vacuum system providing the optical access required for operating an ion trap with $^{43}\text{Ca}^+$. We purchased the laser sources needed for the experiments (with wavelength at 423nm and 375 nm (for photoionization of neutral calcium) as well as 395nm, 397nm, 729nm, 854 nm, 866 nm (for exciting singly charged calcium)). The lasers were set up and frequency stabilized to ultra-stable optical cavities which also had to be set up for this purpose. For quantum state detection, the laser

operating at 729nm needed to have a narrow linewidth to resolve the complicated hyperfine structure of the $S_{1/2}$ to $D_{5/2}$ quadrupole transition. Therefore, we modified the Ti:Sapphire laser we had bought by adding an intra-cavity electro-optical modulator to achieve a high servo-bandwidth. The laser was subsequently stabilized to an ultrastable ULE resonator with a finesse of 400000 based on a design pioneered by the group of J. Hall at JILA, Boulder. With this setup, we succeeded in stabilizing the laser linewidth to about 1 Hz on the time scale of a few seconds and to about 20 Hz on the time scale of a minute. For storing the ions, we duplicated the ion trap that had been successfully operating in the $^{40}\text{Ca}^+$ experiments for a couple of years. Loading of $^{43}\text{Ca}^+$ ions was achieved using an isotope-enriched source of calcium. The rather large nuclear spin of $^{43}\text{Ca}^+$ makes Doppler cooling and state detection more complicated than with $^{40}\text{Ca}^+$. Therefore, we had to set up quite a number of electro-optical and acousto-optical devices for frequency-modulating the laser beams. These beams were subsequently recombined, coupled into optical fibers for transporting the light to the optical table housing the ion trap experiment. For detection of the ions, we designed custom-made lenses capable of a good optical resolution both at the detection wavelength of 397nm and at 729nm for single-ion addressing with a focused laser beam on the quadrupole transition.

The first experimental steps for handling $^{43}\text{Ca}^+$ consisted in learning how to Doppler-cool the isotope and in detecting the quantum state of the ions. For the latter, we decided to shelve one of the hyperfine qubit states in the $D_{5/2}$ manifold by applying a π -pulse on the quadrupole transition. This in turn necessitated characterizing the hyperfine structure splitting of the $S_{1/2}$ to $D_{5/2}$ quadrupole transition which was not well known at that time. For this, we performed high-resolution spectroscopy on the quadrupole transition and recorded more than 30 transitions between Zeeman states at different magnetic field strengths [Benhelm2007]. This procedure made it possible to improve the accuracy of the hyperfine structure constants of the $D_{5/2}$ state by a factor of 1000 over what was known so far. In addition, we found several interesting transitions that are completely free of any first-order Zeeman shift at fields of a few Gauss. The corresponding Zeeman states might be of interest for encoding quantum information if one decides to work with an optical qubit.

Having achieved control over the quadrupole structure, we implemented sideband cooling to the ground state and coherent single-qubit operations on this transition driving the qubit and coupling it to a vibrational mode of the ion (string). We found small heating rates of the vibrational mode of about 370ms per vibrational phonon. For efficient quantum state detection, we

implemented a double pulse technique where two subsequent π -pulses to different $D_{5/2}$ Zeeman states were used for transferring the population of one of the qubit hyperfine ground states. This allows for a population transfer of better than 99% and a quantum state detection with a fidelity of about 99%. For single qubit gates, we find a fidelity on the order of 99%. Initialization of the qubit in the hyperfine clock states with $m_f=0$ required transferring the qubit from the $m_f=4$ that was populated after optical pumping. We evaluated different techniques and decided to use a coherent transfer mediated by the $D_{5/2}$ state since excitation of the quadrupole transitions makes it possible to change the magnetic quantum number m by $\Delta m=\pm 2$ in each step (microwave transfer would require four steps instead of 2). With this procedure, we were able to initialize the qubit with a fidelity of 99%. Next, we investigated the coherence of the hyperfine clock state qubit using microwave and Raman excitation. For a magnetic field of 3.4 G, the fringe amplitude of a Ramsey experiments is still 90% after 100ms excitation time. Lowering the magnetic field to 0.5 Gauss, we observed of several seconds (88% fringe amplitude after 1s) [Benhelm2008]. Using the Raman beams, we have also implemented sideband cooling and coupling to the vibrational mode. These steps have not been analyzed in details so far and will have to be further optimized in the future. For the realization of a two-qubit entangling gate, we studied bichromatic excitation schemes on the quadrupole transition and implemented a high-fidelity gate operation as will be explained further below.

4.2 Segmented ion traps

Working with larger number of ions will become more and more difficult if the ions are stored as a single long ion crystal since the coupling strength between the qubit transition and a vibrational sideband goes down and the vibration mode spectrum becomes more complicated with increasing ion number. This consideration has convinced many ion trapping groups of the necessity of constructing segmented miniaturized ion traps that allow for storing the ions in several locations and for shuttling and recombination of ions held in different segments. In Innsbruck, we have started the construction and operation of segmented ion traps. Progress towards these goals has been slower than expected which is related to our lack of in-house trap fabrication facilities. Currently, we are in cooperating with several companies and research institutions for the development of segmented traps. At the moment, we are operating planar ion traps based in printed circuit board technology that will allow us to learn the basic routines of shuttling

and ion crystal splitting that will also have to be employed once more sophisticated and smaller ion traps become operational.

4.3 Coherent entangling quantum operations

The following subsections describe important results that have been achieved with the $^{40}\text{Ca}^+$ experiment that was operated in parallel to the experiments with $^{40}\text{Ca}^+$ ions. During the time of this grant, the $^{40}\text{Ca}^+$ experiment was technically improved in a number of ways. We replaced the frequency stabilization setup of the 729 qubit laser by an improved design that yielded short-term laser linewidth of about 2 Hz and laser lines of about 40 Hz on the time-scale of a minute. The active magnetic field stabilization was replaced by passive μ -metal shield that now encloses the whole vacuum system. The combination of improved laser frequency and magnetic field stability has improved the coherence time from about 1 ms to 10 ms. In addition, laser sources were replaced by lasers operating more reliably and the setup was changed in such a way that the experiment can now be operated completely autonomously (before, laser sources, detection and computer control were shared with a second experiment investigating cavity-QED effects). Also, we implemented improved procedures that make the coherent operations more reliable. For example, addressing errors can be significantly reduced by decomposing single qubit rotations into several pulses that interleave partial flip operations of the qubit with π -phase shifts. Another example is the use of amplitude-shaped pulses instead of square pulses in order to reduce spurious excitation of unwanted transitions.

4.3.1 Creation of Bell states and their tomographic state analysis

We succeeded in creating all four Bell states $SS\pm DD$, $SD\pm DS$ using a simple sequence of blue sideband pulses coupling the qubit to the vibrational state and single qubit rotations. More importantly, we implemented quantum state tomography for the analysis of the created state. Fig. 1 (a)-(d) shows the real and imaginary parts of the produced density matrices of the four Bell states [Roos2004]. The states are reconstructed by representing their density matrix in an operator basis composed of tensor products of Pauli spin matrices. Measuring the density matrix then amounts to measuring the expectation values of these operators. This is achieved by applying different

(optional) $\pi/2$ pulses to the ions and subsequently detecting their joint quantum state. A complication arises with this procedure as it necessarily requires measurements of observables that do not commute. Since the expectation values are always estimated from a finite number of measurements, quantum

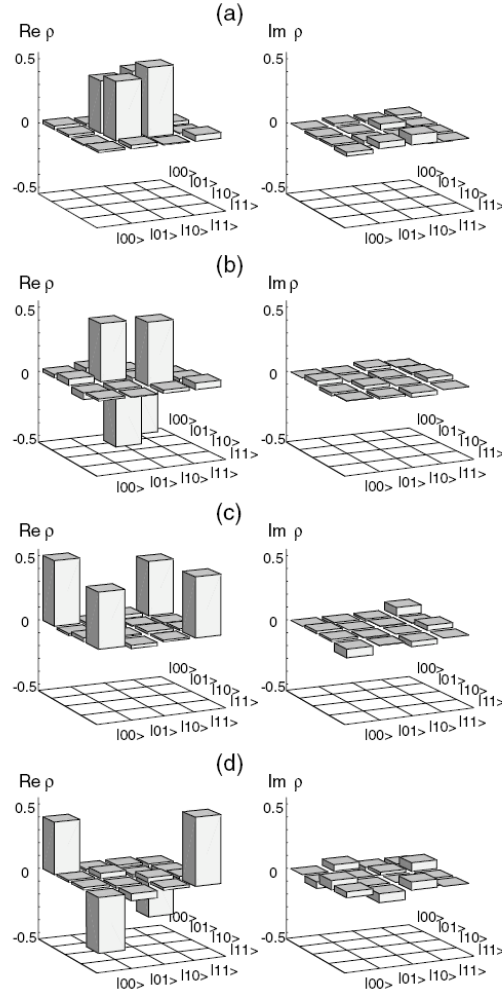


Figure 1: Real and imaginary parts of the Bell states (a) SD+DS, (b) SD-DS, (c) SS+DD, and (d) SS-DD. The fidelity of the produced states ranges between 88% and 91%.

projection noise can give rise to unphysical density matrices if a direct reconstruction technique is applied. This problem can be circumvented by using a maximum likelihood technique that combines all measurement results to find the density matrix that has the highest probability of yielding the observed results.

4.3.2 Long-lived entanglement in decoherence-free subspace

A Bell state prepared in a state $SD \pm DS$ is insensitive against collective phase noise which represents the most important source of decoherence in our experiments. Therefore, we can expect that this state has a lifetime much longer than states $SS \pm DD$ and even single qubit states $S \pm D$. We measured the lifetime of these states by a measurement technique capable of distinguishing the singlet state from the triplet state $SD + DS$. For this, we applied a magnetic field gradient along the ion string that gave rise to a continuous phase evolution of the Bell states as a function of time. Applying collective $\pi/2$ pulses followed by a measurement of the parity $P = p_{SS} + p_{DD} - p_{SD} - p_{DS}$, we recorded parity oscillations as function of time whose contrast could be related to the fidelity of the Bell state [Roos2004]. Using this technique, we could show that the Bell state $SD + DS$ had a lifetime of 1.05 (0.15) s which is consistent with the assumption that the lifetime of the state was only limited by spontaneous decay of the metastable state (lifetime 1.16s).

In order to further increase the lifetime of the entangled state, we transferred the $D_{5/2}$ state population into another Zeeman $S_{1/2}$ ground state and repeated the experiment. In addition, the state was analyzed tomographically. In this decoherence-free subspace, we observed entanglement that lasted for more than 20 s [Häffner2005].

4.3.3 Control and measurement of different three-ion entangled states

We have created three-ion W-states $DDS + DSD + SDD$ as well as a three-ion GHZ $SSS + DDD$ states by using carrier and blue sideband pulses in combination with a CNOT gate operation. These states were analyzed by quantum state tomography. As many quantum algorithms require coherent operations conditioned on measurement results, we demonstrated partial measurements on these states followed by single qubit operations conditioned on the measurement outcome. In these experiments, qubits that are not measured are ‘hidden’ in a superposition of Zeeman states of the metastable level. Then, a qubit is read out and the hidden qubit is again mapped onto a superposition of S and D. we could show how the choice of the measurement basis influenced the produced quantum state. If the measurement is chosen to coincide with the energy state basis S,D, measurement of a single qubit will completely destroy the coherence of the GHZ state but partially preserve coherence in the W-state which is either mapped onto a state DD or onto a state DS+SD. If, however, the

measurement basis is chosen to be $S\pm D$, the GHZ state will be projected onto either the state $SD+DS$ or $SD-DS$. Applying a π -phase shift operation to one of the ions iff the state $SD+DS$ is produced, deterministically transforms the GHZ state into the singlet state $SD-DS$ as we could show in a publication that appeared in Science in 2004 [Roos2004b]. Fig. 2 shows the three-ion density matrix before and after application of the conditional operation.

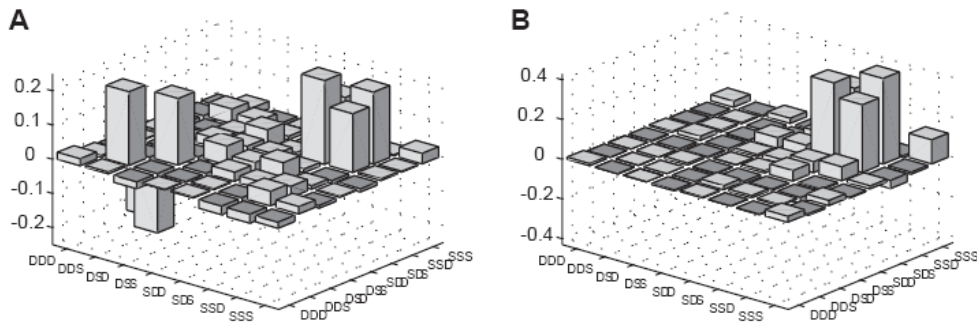


Figure 2: One qubit of the three-ion GHZ state $SSS+DDD$ is measured in the basis $S\pm D$. Panel (A) shows the resulting three-ion density matrix. It consists of an incoherent mixture of the Bell states $SD+DS$ and $SD-DS$. If the relative phase of the Bell state is shifted by π depending on the outcome of the measurement, the state can be deterministically mapped onto a two-ion singlet state as shown in panel (B). This operation constitutes an example of a quantum eraser.

4.3.4 Deterministic quantum teleportation

Teleportation of a quantum state encompasses the complete transfer of information from one particle to another. The complete specification of the quantum state of a system generally requires an infinite amount of information, even for simple two-level systems. Moreover, the principles of quantum mechanics dictate that any measurement on a system immediately alters its state, while yielding at most one bit of information. However, it has been shown that the entangling properties of quantum mechanics, in combination with classical communication, allow quantum-state teleportation to be performed. We succeeded in demonstrating deterministic quantum-state teleportation between a pair of trapped calcium ions. Following closely the original proposal, we create a highly entangled pair of ions and perform a complete Bell-state measurement involving one ion from this pair and a third source ion. State reconstruction conditioned on this measurement is then performed on the other half of the entangled pair using

a procedure already applied for the deterministic transformation of GHZ states in Bell pairs. The measured fidelity is 75%, demonstrating unequivocally the quantum nature of the process [Riebe2004]. Fig. 3 shows a schematic representation of the teleportation process.

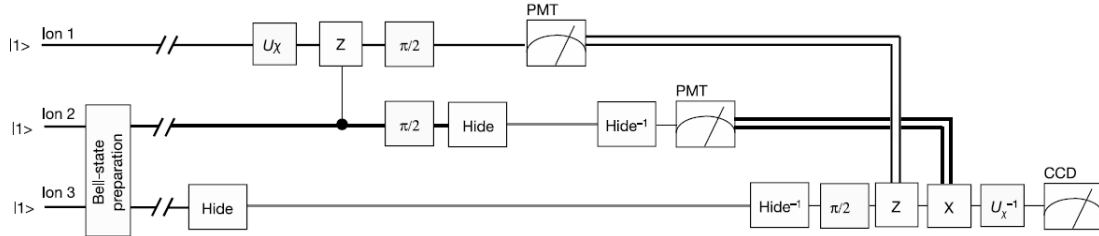


Figure 3: Teleportation from ion 1 to ion 3. A Bell state of ions 2 and 3 is prepared as a resource. The state to be teleported is encoded in ion 1 by the operation U_X . The Bell state analyser consists of a controlled Z-gate followed by $\pi/2$ rotations and a state detection of ions 1 and 2. Note that this implementation uses a Bell basis rotated by $\pi/4$ with respect to the standard notation. Therefore a $\pi/2$ rotation on ion 3 is required before the reconstruction operations Z and X. The latter operations are realized by a π rotation around the z and x axes, respectively. Grey lines indicate qubits that are protected against light scattering. Ions 1 and 2 are detected by observing their fluorescence on a photo-multiplier tube (PMT). Only on a detection event $|0\rangle=D$ is the corresponding reconstruction operation applied to ion 3. Classical information is represented by double lines. For the fidelity analysis we apply $\text{inv}(U_X)$; and measure the quantum state of ion 3 by observing its resonance fluorescence using a CCD camera

In a later experiment, we improved the fidelity of the coherent operations and repeated the teleportation algorithm [Riebe2007]. This time, the teleportation procedure was analyzed using quantum process tomography (see subsection 2.3.6). We succeeded in increasing the teleportation fidelity from 75% to 83%. The process tomography result shows that the main source of infidelity is decoherence while systematic errors are negligible.

4.3.5 Creation and analysis of 8-ion entangled W-states

The creation of scalable multiparticle entanglement demands a non-exponential scaling of resources with particle number. Among the various kinds of entangled states, the ‘W state’ $W=(DSS\dots S+SDS\dots S+\dots+SS\dots D)$ plays an important role as its entanglement is maximally persistent and robust even under particle loss. Such states are central as a resource in quantum information processing and multiparty quantum communication. We investigated the scalable and deterministic generation of four-, five-, six-,

seven- and eight-particle entangled states of the W type with trapped ions. We obtained the maximum possible information on these states by performing full characterization via state tomography, using individual control and detection of the ions. A detailed analysis proved that the entanglement was genuine. The availability of such multi-particle entangled states, together with full information in the form of their density matrices, creates a test-bed for theoretical studies of multi-particle entanglement. Creating W-states of up to eight ions was straight-forward once the procedure had been established for the three-ion case. The difficulty of the experiment was due to the amount of data required for analyzing the state (for the eight ion case, we performed measurements in 6561 different measurement bases and performed measurements on a total of 665,100 entangled states. In this process, we obtained more than 10^6 expectation values of different tensor products of Pauli spin matrices that had to be recombined in a nonlinear way to reconstruct the density matrix.

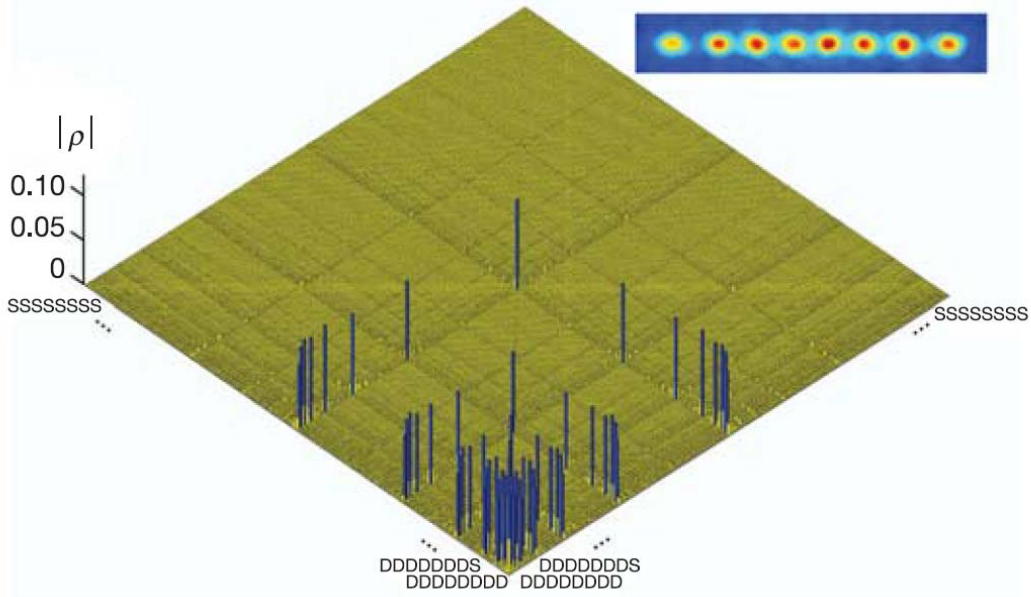


Figure 4: Absolute values, $|\rho|$, of the reconstructed density matrix of a eight-ion W-state as obtained from quantum state tomography. DDDDDDDDS...SSSSSSSS label the entries of the density matrix ρ . Ideally, the blue coloured entries all have the same height of 0.125; the yellow coloured bars indicate noise.

For the analysis of the entanglement of the state, we cooperated with theoreticians of H.Briegel's research group at the Institute for Quantum Optics and Quantum Information in Innsbruck. We were able to show that

the experimentally produced N-ion entangled state shown in Fig. 4 indeed contained genuine N-ion entanglement [Häffner2005b].

4.3.6 Quantum process tomography of a CNOT gate operation

The implementation of high-fidelity entangling gate operations requires a precise knowledge of the error budget. Generally, the implementations of quantum gates are imperfect due to decoherence and various systematic error sources present in experimental setups. A proper description of such an operation which accounts for the possibly non-unitary evolution of the qubits is provided by quantum process tomography. Process tomography has already been applied for characterizing quantum gates in NMR and linear-optics quantum computing. We could show that process tomography constitutes a valuable tool for comparing different ion trap quantum gate implementations and optimizing the experimental parameters [Riebe2006]. This way, we were able to improve our controlled-NOT (CNOT) gate fidelity from 71% to almost 93% (see Fig. 5).

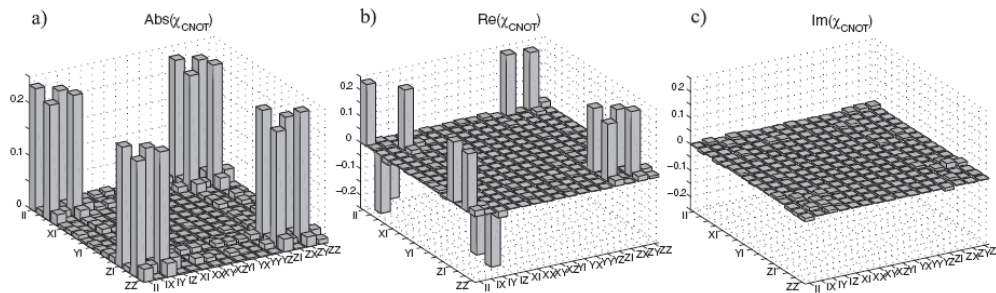


Figure 5: Process matrix obtained by process tomography of a controlled-NOT gate operation. The absolute value, real part, and imaginary part of the gate are shown in (a), (b), and (c), respectively. The matrix is expressed in terms of the products of the identity I and the Pauli operators.

The results convinced us that adiabatically switching on and off laser pulses instead of using rectangular shaped pulses make higher gate speeds possible while preserving high gate fidelities. Thus, this technique and careful optimization of the experimental parameters using process tomography helped to significantly improve the gate fidelity. Moreover, we investigated the action of two successively applied gate operations and compared it to the

predictions from the single gate tomography result. Both results did not completely agree, a result that can probably be explained by assuming that external control parameters fluctuate in a way that violates the Markov assumption of an uncorrelated environment acting independently on the qubits during the first and the second gate operation. The results of the concatenated gate tomography demonstrate the importance of analyzing experimental quantum gate implementations not only as isolated objects but also within a larger gate sequence.

4.3.7 Quantum metrology with entangled states

Atomic clocks rely on the fact that the energy difference between two atomic levels is the same for all atoms of the same kind in the absence of external fields. A measurement of the corresponding atomic oscillation frequency is then used for measuring time. However, electro-magnetic fields are able to slightly shift the atomic energies and thereby to introduce errors in clock measurements. For atoms having a single valence electron, an important effect is caused by electric field gradients interacting with the electric quadrupole moment of the metastable state, leading to shifts of the transition frequency on the order of 10^{-14} . The electric quadrupole moment was measured for a number of isotopes by studying the influence of the field gradient on the transition frequency. Such measurements require lasers of extreme spectral purity and the absence of other energy shift masking the quadrupole shift.

In our experiment with $^{40}\text{Ca}^+$ -ions, the leading energy shift is caused by magnetic fields that shift all energy levels by an amount 100000 bigger than shift caused by the quadrupole moment. Fluctuating magnetic fields would make measurements with a single atom impossible. By using a pair of entangled atoms, we have been able to create quantum states that are insensitive against these magnetic field fluctuations but sensitive to the quadrupole shift (see Fig.6). Measuring the relative phase of the state versus time, we have succeeded in measuring the value of the electric quadrupole moment of the $D_{5/2}$ -state in calcium with a precision surpassing all previous measurements of quadrupole moments by nearly an order of magnitude [Roos2006]. This technique of preparing a 'super-atom' quantum state consisting of two entangled atoms having the desired properties with respect to external field is also of interest for future experiments measuring the transition frequency between the ground state $|S\rangle$ and the metastable $|D\rangle$ state of $^{40}\text{Ca}^+$.

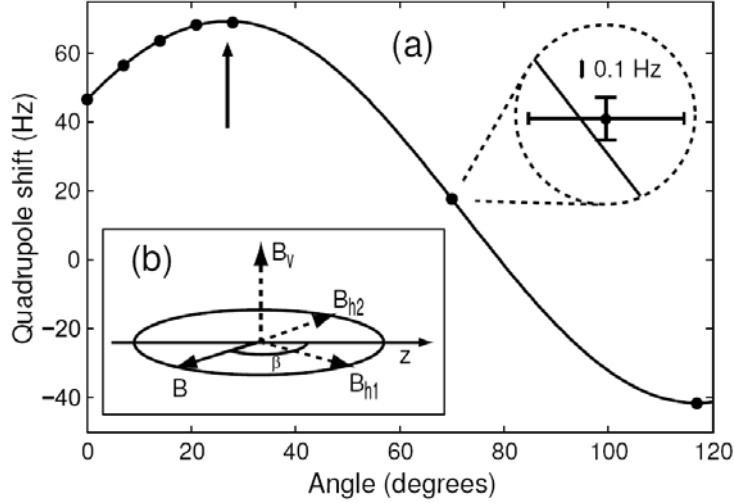


Figure 6: Dependence of the quadrupole shift Δ on the orientation of the magnetic field with respect to the symmetry axis of the ion trap. The measurement is performed with a pair of ions in an entangled state. The atomic quadrupole moment is determined by measuring Δ as a function of the electric field gradient that can be varied by changing the trap oscillation frequency.

4.3.8 Further multi-qubit quantum gates and operations

We succeeded in experimentally realizing and completely characterizing a three qubit quantum Toffoli gate operation that has been implemented for the first time with trapped ions. Our realization is significantly shorter than an implementation based on a decomposition into two-qubit gates. The experimentally realized gate operation was analyzed using quantum process tomography, from which we inferred the gate fidelity. A key element of the experimental implementation was to make use of the availability of more than two states in the ‘quantum bus’ connecting the qubits. This situation is encountered in various quantum computing experiments where the qubits interact with each other via a harmonic oscillator. Up to now, only the lowest two oscillator states had been used in all these experiments as an additional quantum bit. The Toffoli gate experiment demonstrates the advantages of adding an additional third level that enhances the computational power of the system and allows one to find simplified pulse sequences for realizing the gate operation. Quantum process tomography yielded a mean fidelity of 71(3)% for the gate [Monz2008].

We also realized a deterministic entanglement swapping protocol on a string of four ions. As shown in Fig. 7, first ions 1 and 2 as well as ions 3 and 4 are prepared in a Bell state. Then a Bell measurement is performed between ions 1 and 4, and conditioned on the outcome of the measurements, the state of ions 2 and 3 is altered by applying suitable single qubit rotations.

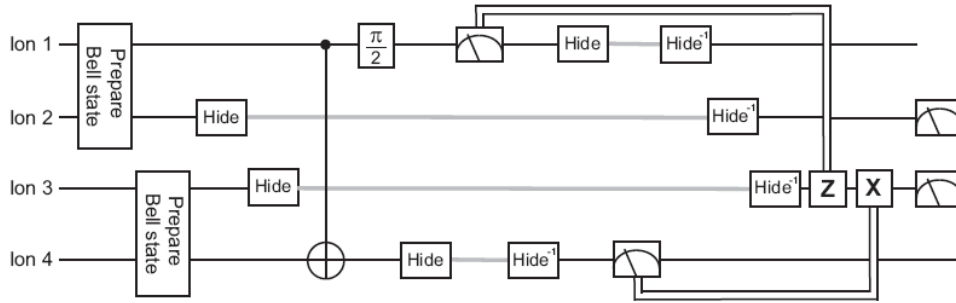


Figure 7: Entanglement swapping protocol implemented in a string of four ions.

For each Bell measurement result, we observe ions 2 and 3 to be in the entangled state with fidelities ranging between 76% and 91%, and an entanglement of formation between 0.49 and 0.75 [Riebe2008].

4.3.9 High-fidelity entangling quantum gate operation

In ion traps, entangling gate operations can be realized by a bichromatic pair of laser beams that collectively interact with the ions. We have introduced a new method of modelling the laser-ion interaction that turns out to be superior to standard techniques for the description of gate operations on optical qubits. The treatment allows for a comparison of the performance of gates based on $\sigma_z \otimes \sigma_z$ and $\sigma_\phi \otimes \sigma_\phi$ interactions, with $\sigma_\phi = \cos\phi \sigma_x + \sin\phi \sigma_y$, on optical transitions where the bichromatic laser field can be realized by an amplitude-modulated laser resonant with the qubit transition [Roos2008]. Shaping the amplitude of the bichromatic laser pulse is shown to make the gates more robust against experimental imperfections.

So far, bichromatic quantum gates have been realized only on hyperfine qubits using Raman beams in counterpropagating beam configuration. Recently, we succeeded in implementing a Mølmer-Sørensen type gate operation on a pair of $^{40}\text{Ca}^+$ ions. This gate operation has yielded a record in terms of the quality of Bell states produced by the gate [Benhelm2008b]. We

have observed entangled states with a fidelity of 99.3(1)% (see Fig 8). The gate mechanism does not require cooling to the ground state of the vibrational mode although it improve the gate fidelity. For a thermal state with an average of 17 motional quanta, we were still able to produce entangled states with a fidelity of 96% [Kirchmair2008].

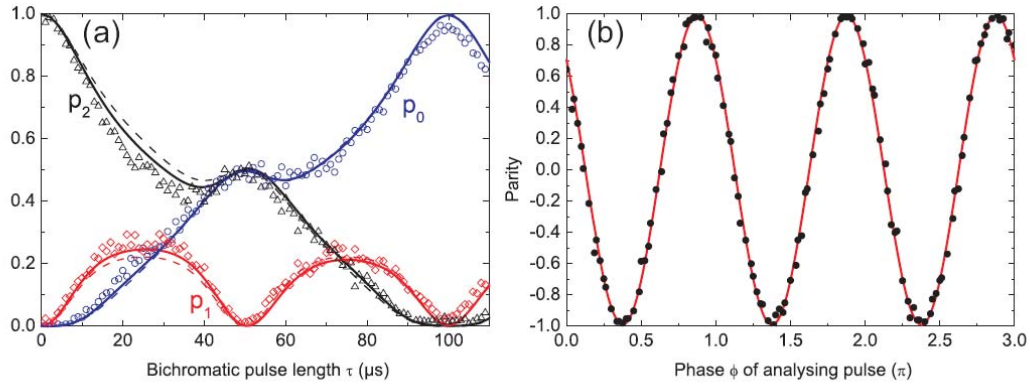


Figure 8: (a) State evolution induced by a Mølmer-Sørensen bichromatic pulse of duration τ . The Rabi frequency $\Omega(t)$ is smoothly switched on and off within 2 μs and adjusted such that a maximally entangled state is created. (b) An $\pi/2$ analysis pulse with phase ϕ applied to both ions prepared in gives rise to a parity oscillation $P(\phi) = \sin(2\phi)$ as a function of ϕ . Combining quantum state data similar to the one shown in panel (a) with the parity oscillation data, we infer a Bell state fidelity of 99.3(1)%.

Differently from previously demonstrated ion trap gates, the current gate operation uses for the first time a bichromatic excitation on an optical qubit, and it thus realizes the Mølmer-Sørensen protocol on a narrow transition such that spontaneous decay from intermediate or neighbouring states does not limit the gate fidelity. Moreover, with this realization gate operations become available with a single travelling wave, no counter-propagating beams are required as for the Raman excitations of previous implementations, and thus the technique allows for a rather simple, robust and straightforward gate operation. In addition, the use of a narrow optical transition minimizes spontaneous scattering events that often limit gate fidelities. The implementation uses in an easy and natural way the advantages provided by pulse shaping as is well known from previous NMR experiments. Thus, the robustness of the gate operations is greatly enhanced without compromising the speed.

With the current gate implementation available, it was possible to concatenate up to 21 gate operations and still retrieve a final maximally entangled (Bell) state with a fidelity of about 80%. This proves the applicability to more extended computations.

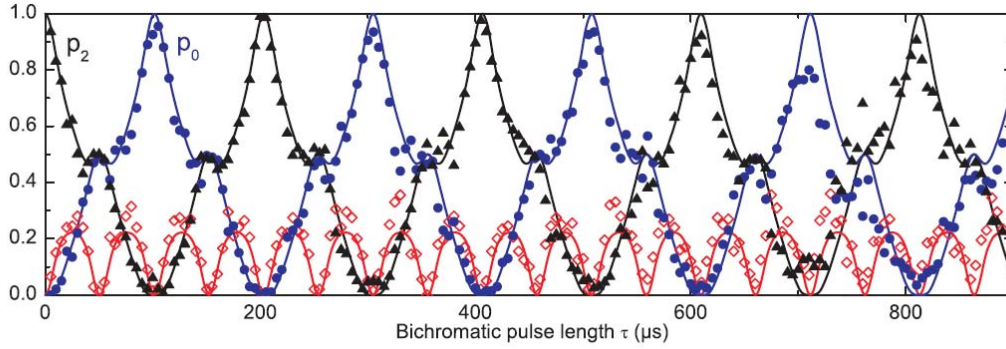


Figure 9: Entanglement dynamics of multiple gate operations. For odd multiples of $50\mu\text{s}$, the gate acting on the initial state SS created an entangled state whose fidelity can be analyzed to obtain information about error sources limiting the gate fidelity of the single gate operation.

The newly demonstrated gate operation opens the door to versatile multi-qubit entangling operations in the way nuclear magnetic resonance experiments are performed: The laser-ion interaction creates entangling interactions between all ions in a string that can be considered the equivalent of spin-spin interactions in NMR experiments. By combining the entangling pulses with the individual addressing available in our Innsbruck experiments, it will be possible to selectively refocus unwanted interactions and keep the desired ones as demonstrated in NMR. In this way the advantage of ion trap experiment of using pure quantum states will be combined with the powerful NMR techniques of state manipulation. Now for the first time we can think of realistic error correction protocols that do not rely on physically separating the qubits.

The gate was realized in the new $^{43}\text{Ca}^+$ setup, however, using a pair of $^{43}\text{Ca}^+$ ions. It should be also applicable to $^{43}\text{Ca}^+$, and we are currently working on implementing the gate operation on the odd isotope.

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